$$\boxed{\Delta W_m = \frac{1}{2}\mu H^2 \Delta V}$$

The magnetostatic energy density w_m is defined as

$$w_m = \lim_{\Delta V \to 0} \frac{\Delta W_m}{\Delta V} = \frac{1}{2}\mu H^2$$

Hence $$w_m = \frac{1}{2}\mu H^2 = \frac{1}{2} B \cdot H = \frac{B^2}{2\mu}$$

$$\boxed{w_m = \frac{B^2}{2\mu}}$$

Thus the energy in a magnetostatic field in a dielectric

$$W_m = \int w_m dV$$

$$\boxed{W_m = \frac{1}{2}\int B \cdot H \, dV = \frac{1}{2}\int \mu H^2 dV}$$

~~Force on magnetic Materials~~

Magnetic Circuits :

we defined magnetomotive force (mmf) $\mathcal{F}$ as

$$\mathcal{F} = NI = \oint H \cdot dl$$

...ductance

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In this case the conductor is assumed to

Convection current does not involve conduction and also does not satisfy ohm's law. This current is occur when current is through an insulating medium such as liquid, rarefied gas or

Convection current density:

When the conductor is uniform area of cross section, and when its a steady current is passes through the conductor then current density is uniform but current density is different at different point if the conductor is of non uniform cross section.

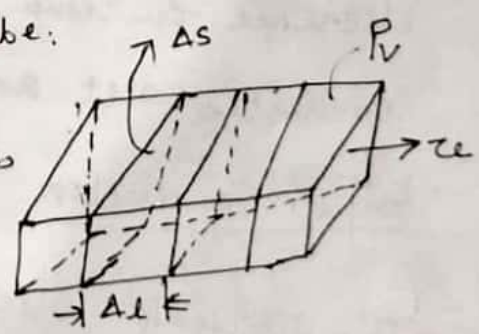
So current density (J) is defined as the current flowing per unit area when the area is held normal to the flow of charge.

$$J = \frac{\Delta I}{\Delta S}$$

$$dI = J \cdot ds$$

$$\boxed{I = \oint J \cdot ds}$$

Ex- A beam of electron in vacuum tube:



Let us consider a filament. If there is a flow of charge of charge density ρ_v at velocity u, the current through the filament is

$$\Delta I = \frac{\Delta Q}{\Delta t} = \frac{\Delta Q}{\Delta V} \cdot \frac{\Delta V}{\Delta t}$$

$$= \frac{\Delta Q}{\Delta V} \cdot \frac{\Delta S \, \Delta l}{\Delta t}$$

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Conduction Current :

A conductor is consist of a large amount of free electrons. So when a conductor is placed in an electric field then a conduction of current is taken place due to the large amount of free electron.

When an electric field 'E' is applied, the force on electron with charge $(Q = -e)$ is $\vec{F} = QE = -e\vec{E}$.

$$\therefore \vec{F} = -eE \quad \text{—(1)}$$

But in crystalline material, the motion of electron is due to by cont collisions with the thermally excited crystalline lattice structure and a last average velocity is maintained. This velocity is called Drift velocity (v_d).

If the electron with mass (m) is moving in an electric field E, with an average drift velocity (v_d).

According to Newton's law, the average change in momentum of the free electron must match the applied force.

$\frac{dp}{dt} = F$ (force) → change in momentum

Thus $\frac{mv_d}{\tau} = -eE$

$$\boxed{v_d = -\frac{e\tau E}{m}}$$

$\boxed{v_d \propto E}$ where τ is the avg. time between collision.

This indicates that the drift velocity of the electron is directly proportional to the applied field.

If there are n electron per unit volume, the electric charge density is given by $\boxed{\rho_v = -ne}$

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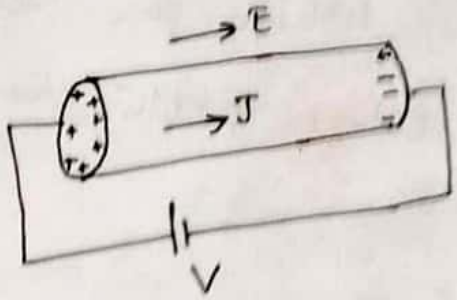
Point form of Ohm's law:



We know that Potential difference but the conductor is directly Proportional to the ~~potential~~ current ~~difference across~~ carrying conductor at a const temp.

$V \propto I \quad V = IR$

$$I = \frac{V}{R} = \frac{V}{\rho \frac{l}{A}} = \frac{VA}{\rho l}$$

$$\Rightarrow \frac{I}{A} = \frac{1}{\rho}\left(\frac{V}{l}\right) \qquad E = \frac{V}{l}$$

$$\boxed{J = \sigma E} \rightarrow$$ which is the Point form of ohm's law

The dir^n of current density vector is same as electric field

If the field are not uniform, $R = \frac{V}{I} = \frac{\int E \cdot dl}{\int J \cdot ds} = \frac{\int E \cdot dl}{\int \sigma E \cdot d}$

If the field are uniform, $\frac{V}{I} = \frac{\int E \cdot dl}{\int \sigma E ds} = \frac{E \cdot l}{\sigma E \cdot s} = \frac{l}{\sigma s}$

Lets law:

Power = rate of change of energy $\left(\frac{W}{t}\right)$ = Force × $\frac{\text{distance}}{\text{time}}$ = Force

$\Delta P = \Delta F \times u \qquad \Delta F = \Delta Q E = \rho_v dv E.$

$\Delta P = \rho_v dv E u \qquad P = \int \rho_v dv E u = \int J E \, dv \qquad J = \rho_v u$

$$\boxed{P = \int J \cdot E \, dv} \rightarrow$$ for non uniform.

Power density $(P_D) = \frac{dP}{dv} = E \cdot J = E \cdot \sigma E = \sigma E^2.$ $\boxed{w_p = \sigma E^2}$

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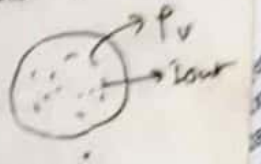
Continuity Equation :

The Principle of Charge Conservation states that charge can be neither created nor destroyed, although equal amount of positive charge and negative may be created (obtain by separation) and destroyed (lost by recombination).

Due to this Principle of charge conservation, the time rate of decrease of charge within a given volume must be equal to the net outward current flow through the closed surface of the volume.



Thus current I_{out} coming out the closed surface is

$$I_{out} = \oint J \cdot ds = -\frac{dQ_i}{dt}$$ where Q_i is the total charge enclosed by the closed surface.

$$-\frac{dQ_i}{dt} = \oint_s J \cdot ds = \int_v (\nabla \cdot J)\, dv$$

$Q_i = \int \rho_v dv$

$$\Rightarrow -\frac{d}{dt}\int_v \rho_v dv = \int_v (\nabla \cdot J)\, dv$$

$$\Rightarrow \int_v -\frac{\partial \rho_v}{\partial t}\, dv = \int_v (\nabla \cdot J)\, dv$$

Comparing the both side, $\boxed{\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}}$ → Continuity of current eqn.

For steady currents, $\frac{\partial \rho_v}{\partial t} = 0$,

$$\boxed{\nabla \cdot J = 0}$$

* $\frac{\partial \rho_v}{\partial t}$ → derivative mean all the charge within the zero.

which show that the total charge leaving a volume is the same as the total charge entering it, that mean there is no accumulation of charge at any point which is the principle of conservation of charge.

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Relaxation Time:

Now we will discuss the continuity eqn and the properties of $\sigma \times \epsilon$ of material for this we introduced charge at some interior point of a material (conductor or dielectric)

From ohm's law $J = \sigma E$. $(\nabla \cdot D = \rho_v)$

and from Gauss law $\nabla \cdot E = \frac{\rho_v}{\epsilon}$

But we know from the continuity eqn

$$\nabla \cdot J = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \nabla \cdot \sigma E = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma(\nabla \cdot E) = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \sigma\left(\frac{\rho_v}{\epsilon}\right) = -\frac{\partial \rho_v}{\partial t}$$

$$\Rightarrow \frac{\partial \rho_v}{\partial t} + \frac{\sigma}{\epsilon}\rho_v = 0$$

$$\Rightarrow \frac{\partial \rho_v}{\rho_v} = -\frac{\sigma}{\epsilon} dt$$

$$\Rightarrow \int \frac{\partial \rho_v}{\rho_v} = -\int \frac{\sigma}{\epsilon} dt \Rightarrow \ln \rho_v = -\frac{\sigma}{\epsilon} t + C.$$

At $t = 0$, $\rho_v = \rho_{v0}$ = initial charge density

$$\ln \rho_{v0} = 0 + C.$$

$$\therefore \ln \rho_v = -\frac{\sigma}{\epsilon} t + \ln \rho_{v0} \Rightarrow \ln \frac{\rho_v}{\rho_{v0}} = -\frac{\sigma}{\epsilon} t$$

$$\boxed{\rho_v = \rho_{v0}\, e^{-\frac{\sigma}{\epsilon} t}}$$

$$\boxed{\rho_v = \rho_{v0}\, e^{-t/T_r}} \quad \text{where } T_r = \frac{\epsilon}{\sigma}$$

= Relaxation time

This eqn shows that as a result of introducing charge at some interior point of the material there is a decay of volume charge density.

When $t = T_r$, $\rho_v = \rho_{v0}\, e^{-1} = 0.37\, \rho_{v0}$

Relaxation time is the time it takes a charge placed in the interior of a material to drop to 37% of its initial value.

* It is a short for good conductor and long for dielectric

For Cu, $T_r = \frac{8.85 \times 10^{-12}}{5.8 \times 10^{7}} = 1.53 \times 10^{-19}$ sec.

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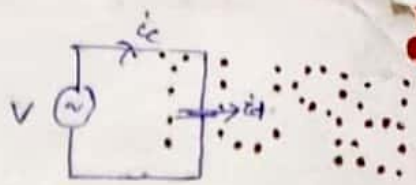
Displacement Current

Consider a parallel plate capacitor connected to an alternating generator of voltage V.



Then, changing current in the conductor is

$$i_c = C\frac{dV}{dt} \qquad C = \frac{\epsilon A}{d} \qquad \epsilon = \epsilon_0\epsilon_r$$

In capacitor, current flow is there when the voltage applied to it is changing. The current through the capacitor is called displacement current. Since there is no conduction current flowing across the capacitor, conduction current is not continuous in the ckt. i.e current flows out of one plate of the capacitor and flows with the other plate.

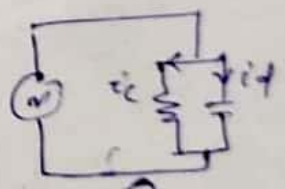
Maxwell's Equations in final form

It is also called Maxwell's Law in Time Varying Case.

Differential form (Point form)	Integral form
$\nabla\cdot D = \rho_v$	$\oint_s D\cdot ds = \int_v \rho_v dv$
$\nabla\cdot B = 0$	$\oint B\cdot ds = 0$
$\nabla\times E = -\frac{\partial B}{\partial t}$	$\oint E\cdot dl = -\frac{\partial}{\partial t}\int_s B\cdot ds$
$\nabla\times H = J + \frac{\partial D}{\partial t}$	$\oint H\cdot dl = \int_s \left(J + \frac{\partial D}{\partial t}\right)\cdot ds$

$C = \frac{\epsilon A}{d}$ A → is area of parallel plate cap. d → is distance of separation bet plates

$$i_c = C\frac{dV}{dt} \cancel{= \frac{dQ}{dt}} = C\frac{dV}{dt}$$



$$i_c = \frac{\epsilon A}{d}\frac{dV}{dt}$$

The electric field inside the capacitor normal to the plates is given by

$$E = \frac{V}{d} \Rightarrow V = Ed.$$

$$i_c = \frac{\epsilon A}{d}\cdot\frac{dV}{dt} = \frac{\epsilon A}{d}\frac{d(Ed)}{dt} = \epsilon A\frac{dE}{dt}$$

$$\frac{i_c}{A} = \epsilon\frac{dE}{dt} = \frac{d}{dt}(\epsilon E) = \frac{dD}{dt}$$

$$\Rightarrow \frac{i_c}{A} = \frac{dD}{dt} \Rightarrow J_d = \frac{dD}{dt}$$

$\therefore i_d \Rightarrow i_d = i_c$

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Transformer and Motional Emfs:

For a circuit with a single turn (N=1) then

$$V_{emf} = -\frac{d\psi}{dt} \qquad \psi = \int B\,ds$$

$$V_{emf} = \oint_L E\cdot dl = -\frac{d}{dt}\int_S B\,ds$$

From the above eqn, it is clear that the variation of flux with time may be caused in three ways

1. By having a stationary loop in a time-varying B field.
2. By having a time-varying loop area in a static B field.
3. By having a time-varying loop area in a time-varying B-field

1. Stationary loop in time-varying B field (Transformer emf)

In this case a stationary conducting loop is in a time varying magnetic B-field. Becomes

$$V_{emf} = \oint_L E\cdot dl = -\int_S \frac{\partial B}{\partial t}\cdot ds$$

This emf induced by time-varying current in a stationary loop is referred to as transformer emf in power analysis. Since it is due to transformer action.

Apply Stokes theorem $\int_S (\nabla\times E)\cdot ds = -\int_S \frac{\partial B}{\partial t}\cdot ds$

$$\Rightarrow \boxed{\nabla\times E = -\frac{\partial B}{\partial t}} \rightarrow$$

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(B) Moving Loop in ...

When a conducting loop is moving in a static B field, an emf is induced in the loop.

Then the force on a charge moving with uniform velocity u in a magnetic field B is

$$F_m = Q\,u \times B.$$

The motional electric field E_m as

$$E_m = \frac{F_m}{Q} = u \times B.$$

If we consider a conducting loop, moving with uniform velocity u as consisting of a large number of free electrons, the emf induced in the loop is

$$\boxed{V_{emf} = \oint_L E_m \cdot dl = \oint_L (u \times B) \cdot dl}$$

This type of emf is called motional emf or flux cutting emf because it is due to motional action.

→ This type of principle apply electromagnetic such as motors, generator & alternator.

Applying stokes theorem,

$$\oint_S (\nabla \times E_m) \cdot ds = \oint_S [\nabla \times (U \times B)] \cdot ds$$

$$\boxed{\nabla \times E_m = \nabla \times (U \times B)}$$

... Varying field

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Now combining (A) + (B),

Total emf as,

$$\text{Vemf} - \oint_L E \cdot dl = -\int_S \frac{\partial B}{\partial t} \cdot ds + \oint_L (u \times B) \cdot dl$$

$$\Rightarrow \oint (\nabla \times E) \cdot ds = -\int_S \frac{\partial B}{\partial t} \cdot ds + \int_S \nabla \times (u \times B) \cdot ds$$

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t} + \nabla \times (u \times B)}$$

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Displacement Current:

For static EM fields, $\nabla \times H = J$ —— (1)

So $\nabla \cdot (\nabla \times H) = \nabla \cdot J$

$\Rightarrow \nabla \cdot J = 0$ —— (2)

But from continuity eqn of current $\nabla \cdot J = -\frac{\partial \rho_v}{\partial t} \neq 0$. —— (3)

So it is clear that eqn (1) can be true only if $\frac{\partial \rho_v}{\partial t} = 0$. ~~for time varying~~

~~Thus from eqn (2), (3) are incompatible~~

~~Consider~~ This is not true and ~~so eqn (1) must be mod~~ For this for this we can add

~~some term modify (1) to agree (3)~~ a term in eqn (1). It becomes

$$\nabla \times H = J + J_d$$ where J_d is to be determined and defined.

Taking divergence of both sides.

$$\nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot J_d$$

$$0 = \nabla \cdot J + \nabla \cdot J_d$$

$$\nabla \cdot J_d = -\nabla \cdot J = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot D) = \nabla \cdot \frac{\partial D}{\partial t}$$

$$\Rightarrow \boxed{J_d = \frac{\partial D}{\partial t}}$$

Hence $\boxed{\nabla \times H = J + \frac{\partial D}{\partial t}}$ → This is maxwell's eqn for time-varying field.

The term $J_d = \frac{\partial D}{\partial t}$ is known as displacement current

J is the conduction current density ($J = \sigma E$).

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Boundary Conditions:

So far, we have considered the existence of electric field & magnetic field in a homogenous medium. If the field exists in a region consisting of two different media, the condition is that the electric field or magnetic field which can satisfy at the interface separating betn the media are called boundary conditions.

These conditions are helpful in determining the field on one side of the boundary if the field on the other side is known.

Now to determine the boundary condn, we can use maxwell's equations.

$$\oint E \cdot dl = 0 \quad \text{--- (1)}$$

$$\oint D \cdot ds = Q \quad \text{--- (2)}$$

$$\oint B \cdot ds = 0 \quad \text{--- (3)}$$

$$\oint H \cdot dl = I \quad \text{--- (4)}$$

Here we decompose the field E, D, B, and H into two orthogonal components.

$$E = E_t + E_n, \quad D = D_t + D_n$$

$$B = B_t + B_n, \quad H = H_t + H_n$$

where, E_t, E_n are tangential and normal components of electric field intensity, B_t & B_n also tangential and normal component of magnetic flux density.

... in the region

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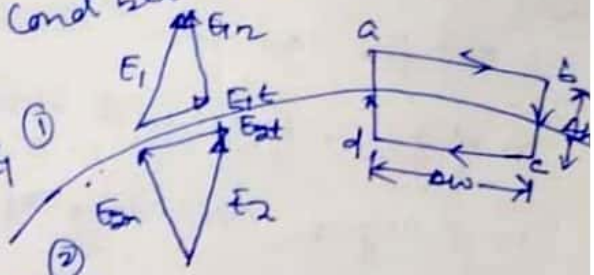
Case ① Dielectric - Dielectric Boundary Condition for Electric field.

Let us consider the boundary cond^n bet^n dielectric - dielectric



Let us consider the E field existing E_1 ① in a region consisting of two different dielectrics characterised ② E_2 by $\epsilon_1 = \epsilon_0 \epsilon_{r_1}$, $\epsilon_2 = \epsilon_0 \epsilon_{r_2}$ as shown in fig.

E_1 and E_2 in media 1 and 2 respectively can be decompose as

$$E_1 = E_{1t} + E_{1n}$$
$$E_2 = E_{2t} + E_{2n}$$

Now to determine the boundary cond^n, we apply maxwell equation. $\oint E \cdot dl = 0$ to the closed path abcda.

$$\int_a^b E \cdot dl + \int_b^c E \cdot dl + \int_c^d E \cdot dl + \int_d^a E \cdot dl = 0$$

$$E_{1t} \Delta w - E_{1n} \frac{\Delta h}{2} - E_{2n} \cdot \frac{\Delta h}{2} - E_{2t} \cdot \Delta w + E_{2n} \cdot \frac{\Delta h}{2} + E_{1n} \cdot \frac{\Delta h}{2}$$

As $\Delta h \to 0$

$$\boxed{E_{1t} = E_{2t}}$$

Thus the tangential components of E are the same on sides of the boundary, in other words Et undergoes no on the boundary and it is said to be continuous across boundary.

Again $D = \epsilon E$

$$E_{1t} = E_{2t}$$

$$\frac{D_{1t}}{\epsilon_1} = \frac{D_{2t}}{\epsilon_2}$$

Dt is said to be discontinuous across interface

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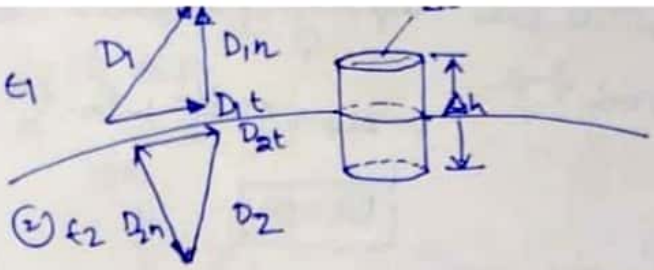
Similarly we can apply GL

$\oint D \cdot ds = Q_{enc}$ to the pillbox (Gaussian surface)



$$\oint_{top} D \cdot ds + \int_{bottom} D \cdot ds + \int_{side} D \cdot ds = Q \quad \text{—— (1)}$$

$$\oint_{\Delta h \to 0} D \cdot ds = 0.$$

$$\Rightarrow D_{1n}\Delta s - D_{2n}\Delta s + 0 = Q$$

$$D_{1n}\Delta s - D_{2n}\Delta s = \rho_s \Delta s$$

$$\boxed{D_{1n} - D_{2n} = \rho_s} \quad \text{—— (2)}$$

where ρ_s is the free charge density at boundary.

If no free charges exists at the interface

i.e. $\rho_s = 0$.

$$D_{1n} - D_{2n} = 0, \quad \boxed{D_{1n} = D_{2n}} \quad \text{—— (3)}$$

Thus the normal component of D is continuous across the interface.

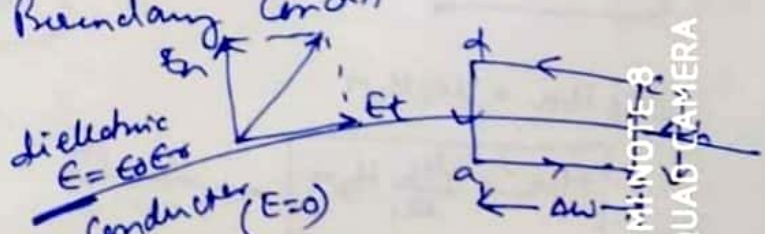
Since $D = \epsilon E$

$$\epsilon_1 E_{1n} = \epsilon_2 E_{2n}$$

$$\boxed{\frac{E_{2n}}{E_{1n}} = \frac{\epsilon_1}{\epsilon_2}}$$

The normal component of $\vec{E}$ is discontinuous at the boundary

Case 2. Conductor - Dielectric Boundary Condn



Here we can assume Conductor to be perfect ($\sigma = \infty$). ($E = 0$).

(copper, silver are perfect conductor)

Now to determine the boundary cond for a conductor we can move same procedure

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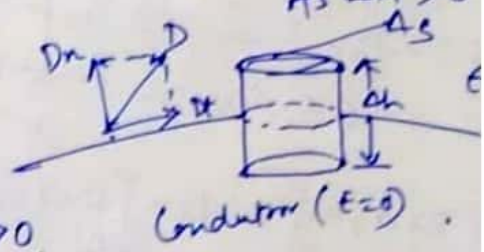
Now we can apply $\oint E \cdot dl = 0$ to the closed path abcda in above figs.

$$0 \cdot \Delta w + 0 \cdot \frac{\Delta h}{2} + E_n \cdot \frac{\Delta h}{2} - E_t \cdot \Delta w - E_n \cdot \frac{\Delta h}{2} - 0 \cdot \frac{\Delta h}{2} = 0$$

As $\Delta h \to 0$

$$\boxed{E_t = 0}$$



Finally we can apply the boundary condn to the pillbox of this fig., $\Delta h \to 0$.

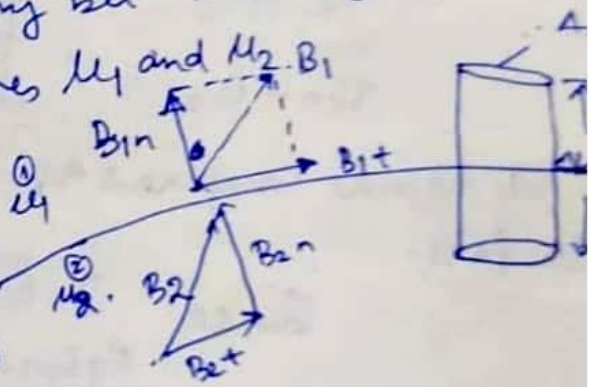
$$D_n \Delta s - 0 \cdot \Delta s = \Delta Q = \rho_s \Delta s$$

$$D_n \Delta s = \rho_s \Delta s \qquad \boxed{D_n = \rho_s} \qquad D_n = \epsilon_0 \epsilon_r E_n$$

$$\boxed{E_n = \frac{\rho_s}{\epsilon_0 \epsilon_r}} \qquad \boxed{D_t = 0}$$

Case-2 Magnetic Boundary Condition.

Let us consider the boundary betn two magnetic media respectively having permitives μ_1 and μ_2.



Now we can apply Gauss law for magnetic field to the pillbox (Gaussian Surface) and take $\Delta h \to 0$

$$B_{1n} \Delta S - B_{2n} \Delta S = 0$$

$$\boxed{B_{1n} = B_{2n}} \quad \text{—— (3)}$$

$$\mu_1 H_{1n} = \mu_2 H_{2n}$$

$$\boxed{H_{1n} = \frac{\mu_2}{\mu_1} H_{2n}} \quad \text{—— (4)}$$

The eqn (3) show that the normal component of B is continuous at the boundary and normal component of H is discontinuous at the boundary.

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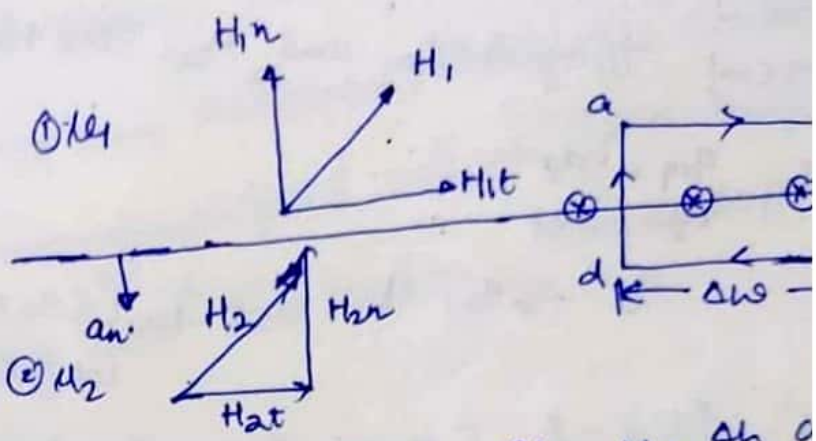
to the closed path abcda where surface current K on the boundary is assumed normal to the path.



$$K\Delta w = H_{1t}\Delta w + H_{1n}\frac{\Delta h}{2} + H_{2n}\cdot\frac{\Delta h}{2} - H_{2t}\cdot\Delta w - H_{2n}\cdot\frac{\Delta h}{2} - H_{1n}\frac{\Delta h}{2}$$

$$K\Delta w = H_{1t}\Delta w - H_{2t}\cdot\Delta w$$

$$\boxed{K = H_{1t} - H_{2t}}$$ → which is show that the tange component of H is discont

$$(\vec{H_1} - \vec{H_2}) \times a_n = \vec{K}$$

$$\boxed{\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K}$$

· If the boundary is free of current or the media conductors (for K is free current density, K=0

$$\boxed{H_{1t} = H_{2t}}$$

$$\boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}}$$

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Power and the Poynting vector.

Since we know that energy can be transported from one point (where a transmitter is located) to another point (with a receiver) by means of EM waves. The rate of such energy transporation can be obtained from maxwell's eqn's.

$$\nabla \times E = -\frac{\partial B}{\partial t} = -\mu \frac{\partial H}{\partial t} \quad \text{—(1)}$$

$$\nabla \times H = J + \frac{\partial D}{\partial t} = \sigma E + \epsilon \frac{\partial E}{\partial t} \quad \text{—(2)}$$

$$E \cdot (\nabla \times H) = J \cdot E + E \cdot \frac{\partial D}{\partial t}$$

(Taking the dot product of each side with E)

$$= \sigma E^2 + E \cdot \epsilon \frac{\partial E}{\partial t} \quad \text{—(3)}$$

and for any vector fields A and B.

$$\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$$

Now Putting $A = H$, $B = E$,

$$\nabla \cdot (H \times E) = E \cdot (\nabla \times H) - H \cdot (\nabla \times E)$$

$$\Rightarrow \quad E \cdot (\nabla \times H) = \nabla \cdot (H \times E) + H \cdot (\nabla \times E) \quad \text{—(4)}$$

from (3) & (4),

$$\nabla \cdot (H \times E) + H \cdot (\nabla \times E) = \sigma E^2 + E \cdot \epsilon \frac{\partial E}{\partial t}$$

$$\Rightarrow -\nabla \cdot (E \times H) + H \cdot \left(-\mu \frac{\partial H}{\partial t}\right) = \sigma E^2 + E \cdot \epsilon \frac{\partial E}{\partial t}$$

$$-\nabla \cdot (E \times H) - \mu H \frac{\partial H}{\partial t} = \sigma E^2 + E \cdot \epsilon \frac{\partial E}{\partial t}$$

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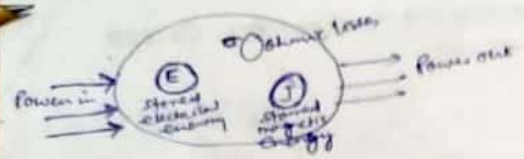

Proof

$$\frac{\partial}{\partial t}H^2 = 2H\frac{\partial H}{\partial t}$$

$$\frac{1}{2}\frac{\partial}{\partial t}H^2 = H\frac{\partial H}{\partial t}$$

$$\frac{1}{2}\frac{\partial}{\partial t}E^2 = E\frac{\partial E}{\partial t}$$

$$-\nabla\cdot(E\times H) - \frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) = \sigma E^2 + \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right)$$

$$\Rightarrow \nabla\cdot(E\times H) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\mu H^2\right) - \frac{\partial}{\partial t}\left(\frac{1}{2}\epsilon E^2\right) - \sigma E^2$$

Taking the volume integral of both sides.

$$\int_V \nabla\cdot(E\times H)\,dV = -\frac{\partial}{\partial t}\int_V\left(\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right)dV - \int_V \sigma E^2\,dV$$

$$\Rightarrow \boxed{\oint_S (E\times H)\cdot ds = -\frac{\partial}{\partial t}\int\left[\frac{1}{2}\epsilon E^2 + \frac{1}{2}\mu H^2\right]dV - \int_V \sigma E^2\,dV} \quad (*)$$

taking single left hand side

Total Power leaving the volume = Rate of decrease in energy stored in electric and magnetic fields - ohmic Power dissipated.

The eqn (*) is referred to as Poynting's theorem

The first term on the right hand side is the rate of decrease in energy stored in the electric and magnetic fields. The 2nd term the power dissipated.

The quantity $E\times H$ on left hand side is known as Poynting vector P.

$$\boxed{P = E\times H}$$

* The Poynting vector is defined as the cross product of the vectors $E\times H$. It gives the magnitude as well as the direction

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In which power flows in time varying electromagnetic fields Poynting vector gives a final answer about the power dissipation in a region. The dir^n of power flow at any point is normal to both E and H vectors. The concept of Poynting vector is used to explain the power flow in a dielectric.

Poynting theorem

Poynting's theorem states that the net power flowing out a given volume V is equal to the time rate of decrease in the energy stored within V minus the conduction losses.

Let us assume that

$$E(z,t) = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

then $$H(z,t) = \frac{E_0}{\eta} e^{-\alpha z} \cos(\omega t - \beta z)\, a_y$$

$$\boxed{P(z,t) = \vec{E} \times \vec{H} = \frac{E_0^2 e^{-2\alpha z}}{\eta} \cos^2(\omega t - \beta z)\, a_z}$$

Case 1 In case of perfect dielectric $\alpha = 0$,

$$P_{z,avg} = \frac{1}{T}\int_0^T P(z,t)\, dt$$

$$= \frac{1}{T}\int_0^T \frac{E_0^2}{\eta} \cos^2(\omega t - \beta z)\, dt$$

$$\left[\frac{1 + \cos 2(\omega t - \beta z)}{2}\right]$$

$$= \frac{E_0^2}{2T\eta}\left[t + \frac{\sin(2\omega t - 2\beta z)}{2\omega}\right]_0^T$$

$$\boxed{P_{z,av} = \frac{1}{2}\frac{E_0^2}{\eta}}$$

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Case 2

For a lossy dielectric,

$$E = E_{x0} e^{-\alpha z} \cos(\omega t - \beta z)\, a_x$$

as $\eta = \eta_m \angle \theta_\eta$ as η is a complex number

$$H = \frac{E_{x0}}{\eta_m} e^{-\alpha z} \cos(\omega t - \beta z - \theta_\eta)$$

$$P_z = E \times H = \frac{E_{x0}^2}{\eta_m} \cos^{-2\alpha z} \cos(\omega t - \beta z) \cos(\omega t - \beta z - \theta_\eta)\, a_z$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$

$$P_z = \frac{1}{2} \frac{E_{x0}^2}{\eta_m} e^{-2\alpha z} \left[\cos(2\omega t - 2\beta z - \theta_\eta) + \cos\theta_\eta\right] a_z$$

$$\boxed{P_{z,avg} = \frac{1}{2} \frac{E_{x0}^2}{\eta_m} e^{-2\alpha z} \cos\theta_\eta\, a_z}$$

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Concept of Retarded Vector Potential.

The time varying potential are usually as retarded potentials. It can be used for application in radiations problems in which the distribution of the source is known approximately. It can be shown that any electromagnetic disturbance is found to travel at a velocity given by

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ through any homogenous medium

described by μ and ϵ. In case of free space, this velocity turns out to be velocity of light 3×10^8 m/sec.

Then the potential at any point is due to the value of the charge density at some distant point at the same instant but to its value at the same previous time.

Since the effect propagates at a finite velocity.

In static field $V = \int_v \frac{[\rho_v]\,dv}{4\pi\epsilon R}$ and $A = \int_v \frac{\mu[J]\,dv}{4\pi R}$

The term $[\rho_v]$ or $[J]$ means the time t in $\rho_v(x,y,z,t)$ or $J(x,y,z,t)$ is replaced by the retarded time t' is given by

$t' = t - \frac{R}{c}$, R is the distance betn source point and observation point.

The potential V and A are respectively called retarded electric potential and retarded magnetic vector potential.

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The scalar electric potential V may be expressed in terms of static charge distribution.

$$V = \int_v \frac{\rho_v dv}{4\pi\epsilon R}$$

and the vector magnetic potential as $A = \int \frac{\mu J dv}{4\pi R}$.

we would like to examine what happens to these potentials when the fields are time varying.

Since $\nabla \cdot B = 0$ where $B = \nabla \times A$. —— (1)

and $E = -\nabla V$ —— (2)

so $\nabla \times E = -\nabla \times \nabla V = 0$

$\Rightarrow \nabla \times E = 0$

But the point form of Faraday's law states that $\nabla \times E$ is not generally zero.

For this we can add an unknown term in eqn (2).

$$E = -\nabla V + N.$$

$$\Rightarrow \nabla \times E = -\nabla \times \nabla V + \nabla \times N$$

$$\Rightarrow \nabla \times E = \nabla \times N$$

$$\Rightarrow \nabla \times N = \nabla \times E = -\frac{\partial B}{\partial t} = -\frac{\partial}{\partial t}(\nabla \times A)$$

$$\Rightarrow \nabla \times N = -\nabla \times \frac{\partial A}{\partial t}$$

The simplest sol^n of the above eqn^ $N = -\frac{\partial A}{\partial t}$

$$\therefore \boxed{E = -\nabla V - \frac{\partial A}{\partial t}} \quad \text{—— (3)}$$

Now taking divergence in the above eqn^ (3)

$$\nabla \cdot E = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla \cdot A)$$

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$$\nabla\cdot\frac{D}{\epsilon} = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla\cdot A)$$

$$\frac{\rho_v}{\epsilon} = -\nabla^2 V - \frac{\partial}{\partial t}(\nabla\cdot A)$$

$$\Rightarrow \boxed{\nabla^2 V + \frac{\partial}{\partial t}(\nabla\cdot A) = -\frac{\rho_v}{\epsilon}} \quad \text{——— (4)}$$

Also we know maxwell's eq-ein.

$$\nabla\times H = J + \frac{\partial D}{\partial t} \qquad B = \mu H$$

$$\Rightarrow \nabla\times\frac{B}{\mu} = J + \frac{\partial D}{\partial t}$$

$$\nabla\times B = \mu J + \mu\epsilon\frac{\partial E}{\partial t}$$

$$= \mu J + \mu\epsilon\frac{\partial}{\partial t}\left[-\nabla V - \frac{\partial A}{\partial t}\right]$$

$$\nabla\times B = \mu J - \mu\epsilon\nabla\frac{\partial V}{\partial t} - \mu\epsilon\frac{\partial^2 A}{\partial t^2}$$

$$\Rightarrow \nabla\times\nabla\times A = \mu J - \mu\epsilon\nabla\frac{\partial V}{\partial t} - \mu\epsilon\frac{\partial^2 A}{\partial t^2}$$

$$\Rightarrow \nabla(\nabla\cdot A) - \nabla^2 A = \mu J - \mu\epsilon\nabla\frac{\partial V}{\partial t} - \mu\epsilon\frac{\partial^2 A}{\partial t^2}$$

$$\Rightarrow \nabla^2 A - \nabla(\nabla\cdot A) = -\mu J + \mu\epsilon\nabla\frac{\partial V}{\partial t} + \mu\epsilon\frac{\partial^2 A}{\partial t^2} \quad \text{——— (5)}$$

If we choose a vector field A in terms of divergence and suppose $\nabla\cdot A = -\mu\epsilon\frac{\partial V}{\partial t}$ which is called Lorentz cond^n for Potential

Then the eq^n (4) & (5) becomes

$$\boxed{\nabla^2 V - \mu\epsilon\frac{\partial^2 V}{\partial t^2} = -\frac{\rho_v}{\epsilon}} \quad \text{——— (6)}$$

$$\boxed{\nabla^2 A - \mu\epsilon\frac{\partial^2 A}{\partial t^2} = -\mu J} \quad \text{——— (7)}$$

These are above two wave equations.

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